Dynamic Burrows-Wheeler Transform

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Motivation

Recent Advances in High-Throughput Sequencing

Index large dynamic texts (genomes) to speed up approximate pattern matching (myriads of short patterns, at most one error).

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Two possible efficient solutions

	FM-Index BWT + Suffix Array Ferragina-Manzini FOCS'00	LZ-Index LZ-factorisation Navarro SPIRE'04
Advantages	Compressed indexes Approximate string matching	
DNA, 50 Mb	19 to 34 MB	44 MB

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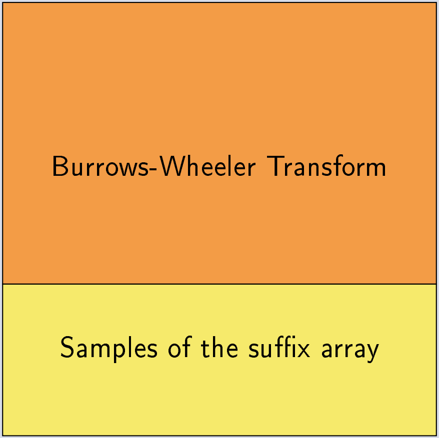
Our main motivation: index large dynamic texts

Single modification in the text → Creation of a new index from scratch?

Our choice

FM-Index: more efficient and probably (?) easier to update



BWT is used to **quickly**:

- retrieve the original text;
- search for patterns.

We need the whole structure;

SA is used to **quickly**:

- find the position of each occurrence of a pattern.

We can sample the SA and recover the missing values.

Burrows-Wheeler Transform of $T = \overset{0}{C}\overset{1}{T}\overset{2}{C}\overset{3}{T}\overset{4}{G}\overset{5}{C}\overset{6}{\$}$

The big picture

The Burrows-Wheeler Transform is a permutation of a text T that eases the compression of T.

Burrows-Wheeler Transform of $T = \begin{array}{ccccccc} 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ C & T & C & T & G & C & \$ \end{array}$

The big picture

The Burrows-Wheeler Transform is a permutation of a text T that eases the compression of T.

unsorted cyclic shifts

	unsorted $T^{[i]}$						
0	C	T	C	T	G	C	$
1	T	C	T	G	C	$	C
2	C	T	G	C	$	C	T
3	T	G	C	$	C	T	C
4	G	C	$	C	T	C	T
5	C	$	C	T	C	T	G
6	$	C	T	C	T	G	C

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3	T	G	C	$	C	T	C
4	G	C	$	C	T	C	T
5	C	$	C	T	C	T	G
6	$	C	T	C	T	G	C

sorted cyclic shifts, BWT

	F ↓	sorted $T^{[i]}$					L ↓
0	$	C	T	C	T	G	C
1	C	$	C	T	C	T	G
2	C	T	C	T	G	C	$
3	C	T	G	C	$	C	T
4	G	C	$	C	T	C	T
5	T	C	T	G	C	$	C
6	T	G	C	$	C	T	C
	↓ sorted						↓ unsorted

Burrows-Wheeler Transform of $T = \begin{array}{ccccccc} 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ C & T & C & T & G & C & \$ \end{array}$

The big picture

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unsorted cyclic shifts

	unsorted $T^{[i]}$						
0	C	T	C	T	G	C	\$
1	T	C	T	G	C	\$	C
2	C	T	G	C	\$	C	T
$\boxed{3}$	T	G	C	\$	C	T	C
4	G	C	\$	C	T	C	T
5	C	\$	C	T	C	T	G
6	\$	C	T	C	T	G	C

sorted cyclic shifts, BWT and suffix array

	F ↓	sorted $T^{[i]}$					*L* ↓	*SA* ↓
0	\$	C	T	C	T	G	C	6
1	C	\$	C	T	C	T	G	5
2	C	T	C	T	G	C	\$	0
3	C	T	G	C	\$	C	T	2
4	G	C	\$	C	T	C	T	4
5	T	C	T	G	C	\$	C	1
6	T	G	C	\$	C	T	C	$\boxed{3}$
	↓ sorted						↓ unsorted	

Burrows-Wheeler Transform of $T = \overset{0}{\text{C}}\,\overset{1}{\text{T}}\,\overset{2}{\text{C}}\,\overset{3}{\text{T}}\,\overset{4}{\text{G}}\,\overset{5}{\text{C}}\,\overset{6}{\$}$

The big picture

The Burrows-Wheeler Transform is a permutation of a text T that eases the compression of T.

unsorted cyclic shifts

	unsorted $T^{[i]}$						
0	C	T	C	T	G	C	\$
1	T	C	T	G	C	\$	C
2	C	T	G	C	\$	C	T
$\boxed{3}$	T	G	C	\$	C	T	C
4	G	C	\$	C	T	C	T
5	C	\$	C	T	C	T	G
6	\$	C	T	C	T	G	C

sorted cyclic shifts, BWT and suffix array

	F ↓		sorted $T^{[i]}$				L ↓	SA ↓	ISA ↓
0	\$	C	T	C	T	G	C	6	2
1	C	\$	C	T	C	T	G	5	5
2	C	T	C	T	G	C	\$	0	3
3	C	T	G	C	\$	C	T	2	6
4	G	C	\$	C	T	C	T	4	4
5	T	C	T	G	C	\$	C	1	1
6	T	G	C	\$	C	T	C	$\boxed{3}$	0
	↓ sorted						↓ unsorted		

Which column do we have to consider?

First column $F = \$$ C C C G T T

- gathers identical symbols;
- easy to compress $\$r_3\text{CG}r_2\text{T}$;
- recovering T from F is impossible.

Which column do we have to consider?

First column $F = \$ \text{ C C C G T T}$

- gathers identical symbols;
- easy to compress $\$r_3\text{CG}r_2\text{T}$;
- recovering T from F is impossible.

Last column $L = \text{C G } \$ \text{ T T C C}$

- has a tendancy to create packets of identical symbols:
- eases compression;
- is recovering T from L possible?

FM-index: a dynamic data structure?

$$T = \begin{array}{ccccccc} 0&1&2&3&4&5&6\\ \mathrm{C}&\mathrm{T}&\mathrm{C}&\mathrm{T}&\mathrm{G}&\mathrm{C}&\$ \end{array} \quad \rightarrow \quad T' = \begin{array}{cccccccc} 0&1&2&3&4&5&6&7\\ \mathrm{C}&\mathrm{T}&G&\mathrm{C}&\mathrm{T}&\mathrm{G}&\mathrm{C}&\$ \end{array}$$

BWT **Easy (Laurent!)**

$$BWT(\mathrm{CTCTGC\$}) = \mathrm{C\ G\ \$\ T\ T\ C\ C}$$

$$BWT(\mathrm{CT}G\mathrm{CTGC\$}) = \mathrm{C\ G}\ G\ \mathrm{\$\ T\ T\ C\ C}$$

FM-index: a dynamic data structure?

$$T = \begin{array}{ccccccc} 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ \mathrm{C} & \mathrm{T} & \mathrm{C} & \mathrm{T} & \mathrm{G} & \mathrm{C} & \$ \end{array} \quad \rightarrow \quad T' = \begin{array}{cccccccc} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \mathrm{C} & \mathrm{T} & G & \mathrm{C} & \mathrm{T} & \mathrm{G} & \mathrm{C} & \$ \end{array}$$

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$$BWT(\mathrm{CT}G\mathrm{CTGC\$}) = \mathrm{C\ G}\ G\ \mathrm{\$\ T\ T\ C\ C}$$

Suffix Array **Hard (Mikaël!)**

$$SA(\mathrm{CTCTGC\$}) = 6\ 5\ 0\ 2\ 4\ 1\ 3$$

$$SA(\mathrm{CT}G\mathrm{CTGC\$}) = 7\ 6\ 3\ 0\ 5\ 2\ 4\ 1$$

FM-index: a dynamic data structure?

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$$BWT(\mathrm{CTCTGC\$}) = \mathrm{C\ G\ \$\ T\ T\ C\ C}$$
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$$SA(\mathrm{CT}G\mathrm{CTGC\$}) = 7\ 6\ 3\ 0\ 5\ 2\ 4\ 1$$

BWT update looks easier than SA update

- BWT and SA are strongly related;
- BWT update will help for SA update.

FM-index: a dynamic data structure?

$$T = \begin{array}{ccccccc} 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ \mathrm{C} & \mathrm{T} & \mathrm{C} & \mathrm{T} & \mathrm{G} & \mathrm{C} & \$ \end{array} \quad \rightarrow \quad T' = \begin{array}{cccccccc} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \mathrm{C} & \mathrm{T} & G & \mathrm{C} & \mathrm{T} & \mathrm{G} & \mathrm{C} & \$ \end{array}$$

BWT **Easy (Hum, let me think about it!)**

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FM-index: a dynamic data structure?

$$T = \overset{0}{\mathrm{C}}\,\overset{1}{\mathrm{T}}\,\overset{2}{\mathrm{C}}\,\overset{3}{\mathrm{T}}\,\overset{4}{\mathrm{G}}\,\overset{5}{\mathrm{C}}\,\overset{6}{\$} \quad \rightarrow \quad T' = \overset{0}{\mathrm{C}}\,\overset{1}{\mathrm{T}}\,\overset{2}{G}\,\overset{3}{\mathrm{C}}\,\overset{4}{\mathrm{T}}\,\overset{5}{\mathrm{G}}\,\overset{6}{\mathrm{C}}\,\overset{7}{\$}$$

BWT

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$$BWT(\mathrm{CTCTGC\$}) = \mathrm{C\ G\ \$\ T\ T\ C\ C}$$
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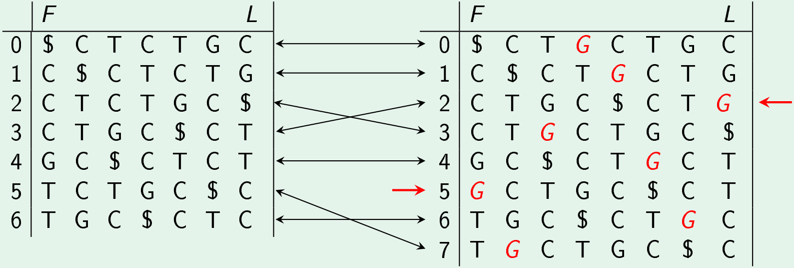
Conceptual matrices



	F						*L*
0	$	C	T	C	T	G	C
1	C	$	C	T	C	T	G
2	C	T	C	T	G	C	$
3	C	T	G	C	$	C	T
4	G	C	$	C	T	C	T
5	T	C	T	G	C	$	C
6	T	G	C	$	C	T	C

	F							*L*
0	$	C	T	*G*	C	T	G	C
1	C	$	C	T	*G*	C	T	G
2	C	T	G	C	$	C	T	*G*
3	C	T	*G*	C	T	G	C	$
4	G	C	$	C	T	*G*	C	T
5	*G*	C	T	G	C	$	C	T
6	T	G	C	$	C	T	*G*	C
7	T	*G*	C	T	G	C	$	C

From $BWT(T)$=CG\$TTCC back to T=CTCTGC\$

Right-to-left reconstruction of T from L

i	L
0	C
1	G
2	\$
3	T
4	T
5	C
6	C

From $BWT(T)$=CG\$TTCC back to T=CTCTGC\$

Right-to-left reconstruction of T from L

i	F	L
0	\$	C
1	C	G
2	C	\$
3	C	T
4	G	T
5	T	C
6	T	C

Deduce sorted F from unsorted L

From $BWT(T)=\text{CG\$TTCC}$ back to $T=\text{CTCTGC\$}$

Right-to-left reconstruction of T from L

i	F	L	LF
0	\$	C	
1	C	G	
2	C	$\boxed{\$}$	
3	C	T	
4	G	T	
5	T	C	
6	T	C	

Right-to-left suffixes of T

$\boxed{\$}$

From $BWT(T)$=CG$TTCC back to T=CTCTGC$

Right-to-left reconstruction of T from L

i	F	L	LF
0	$	C	
1	C	G	
2	C	$\boxed{\$}$	
3	C	T	
4	G	T	
5	T	C	
6	T	C	

Right-to-left suffixes of T

$

From $BWT(T)=\mathrm{CG\$TTCC}$ back to $T=\mathrm{CTCTGC\$}$

Right-to-left reconstruction of T from L

i	F	L	LF
0	$\boxed{\$}$	C	
1	C	G	
2	C	$\boxed{\$}$	
3	C	T	
4	G	T	
5	T	C	
6	T	C	

Right-to-left suffixes of T

$\$$

From $BWT(T)$=CG$TTCC back to T=CTCTGC$

Right-to-left reconstruction of T from L

i	F	L	LF
0	$	C	
1	C	G	
2	C	$	0
3	C	T	
4	G	T	
5	T	C	
6	T	C	

Right-to-left suffixes of T

$

From $BWT(T)$=CG\$TTCC back to T=CTCTGC\$

Right-to-left reconstruction of T from L

i	F	L	LF
0	$\boxed{\$}$	C	
1	C	G	
2	C	\$	0
3	C	T	
4	G	T	
5	T	C	
6	T	C	

Right-to-left suffixes of T

\$

From $BWT(T)$=CG\$TTCC back to T=CTCTGC\$

Right-to-left reconstruction of T from L

i	F	L	LF
0	\$	$\boxed{\text{C}}$	
1	C	G	
2	C	\$	0
3	C	T	
4	G	T	
5	T	C	
6	T	C	

Right-to-left suffixes of T

$\boxed{\text{C}}$ \$

From $BWT(T)$=CG$TTCC back to T=CTCTGC$

Right-to-left reconstruction of T from L

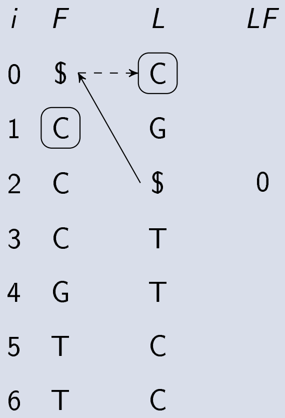
i	F	L	LF
0	$	$\boxed{\text{C}}$	
1	C	G	
2	C	$	0
3	C	T	
4	G	T	
5	T	C	
6	T	C	

Right-to-left suffixes of T

C $

From $BWT(T)$=CG$TTCC back to T=CTCTGC$

Right-to-left reconstruction of T from L



i	F	L	LF
0	$	C	
1	C	G	
2	C	$	0
3	C	T	
4	G	T	
5	T	C	
6	T	C	

Right-to-left suffixes of T

C $

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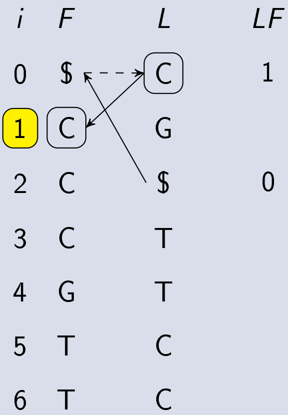
Right-to-left reconstruction of T from L



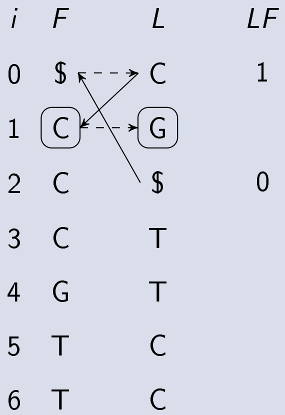
i	F	L	LF
0	\$	C	1
1	C	G	
2	C	\$	0
3	C	T	
4	G	T	
5	T	C	
6	T	C	

Right-to-left suffixes of T

C \$

From $BWT(T)$=CG$TTCC back to T=CTCTGC$

Right-to-left reconstruction of T from L



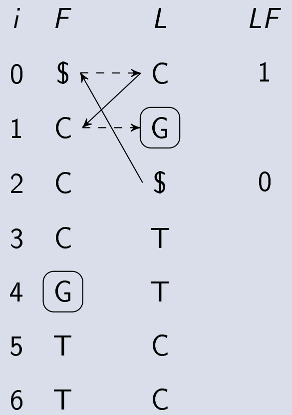
i	F	L	LF
0	$	C	1
1	$\boxed{\text{C}}$	$\boxed{\text{G}}$	
2	C	$	0
3	C	T	
4	G	T	
5	T	C	
6	T	C	

Right-to-left suffixes of T

$\boxed{\text{G}}$C $

From $BWT(T)=\mathrm{CG\$TTCC}$ back to $T=\mathrm{CTCTGC\$}$

Right-to-left reconstruction of T from L



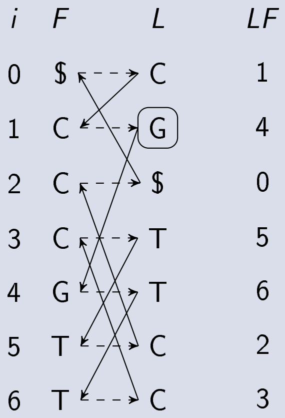
i	F	L	LF
0	\$	C	1
1	C	$\boxed{G}$	
2	C	\$	0
3	C	T	
4	$\boxed{G}$	T	
5	T	C	
6	T	C	

Reconstructed text T

G C \$

From $BWT(T)=\text{CG\$TTCC}$ back to $T=\text{CTCTGC\$}$

Right-to-left reconstruction of T from L



i	F	L	LF
0	\$	C	1
1	C	$\boxed{\text{G}}$	4
2	C	\$	0
3	C	T	5
4	G	T	6
5	T	C	2
6	T	C	3

Reconstructed text T

C T C T G C \$

A visual representation of *LF* for $T = \overset{0}{C}\overset{1}{T}\overset{2}{C}\overset{3}{T}\overset{4}{G}\overset{5}{C}\overset{6}{\$}$

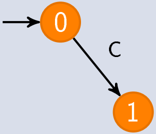
Considering *LF* as an automaton

i	F	L	LF
0	\$	C	1
1	C	G	4
2	C	\$	0
3	C	T	5
4	G	T	6
5	T	C	2
6	T	C	3

A visual representation of LF for $T = \begin{array}{ccccccc} 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ \mathrm{C} & \mathrm{T} & \mathrm{C} & \mathrm{T} & \mathrm{G} & \mathrm{C} & \$ \end{array}$

Considering *LF* as an automaton

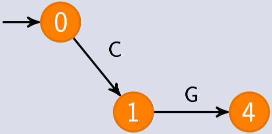
i	*F*	*L*	*LF*
0	$	C	1
1	C	G	4
2	C	$	0
3	C	T	5
4	G	T	6
5	T	C	2
6	T	C	3



A visual representation of *LF* for $T = \overset{0}{\text{C}}\,\overset{1}{\text{T}}\,\overset{2}{\text{C}}\,\overset{3}{\text{T}}\,\overset{4}{\text{G}}\,\overset{5}{\text{C}}\,\overset{6}{\text{\$}}$

Considering *LF* as an automaton

i	*F*	*L*	*LF*
0	$	C	1
1	C	G	4
2	C	$	0
3	C	T	5
4	G	T	6
5	T	C	2
6	T	C	3



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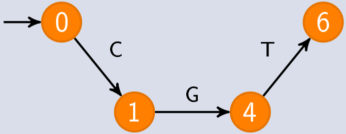
Considering *LF* as an automaton

i	F	L	LF
0	\$	C	1
1	C	G	4
2	C	\$	0
3	C	T	5
4	G	T	6
5	T	C	2
6	T	C	3



A visual representation of *LF* for $T = \overset{0}{C}\overset{1}{T}\overset{2}{C}\overset{3}{T}\overset{4}{G}\overset{5}{C}\overset{6}{\$}$

Considering *LF* as an automaton

i	*F*	*L*	*LF*
0	$	C	1
1	C	G	4
2	C	$	0
3	C	T	5
4	G	T	6
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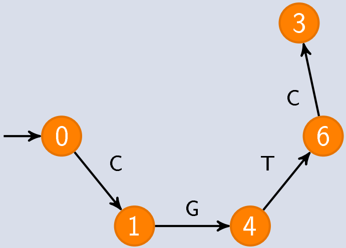
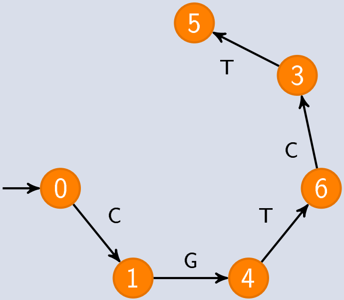
6	T	C	3



A visual representation of *LF* for $T = \overset{0}{C}\,\overset{1}{T}\,\overset{2}{C}\,\overset{3}{T}\,\overset{4}{G}\,\overset{5}{C}\,\overset{6}{\$}$

Considering *LF* as an automaton

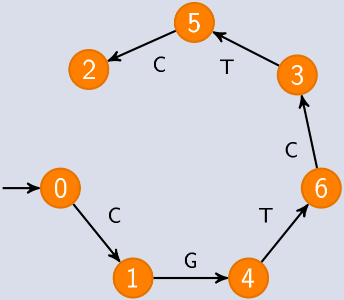
i	*F*	*L*	*LF*
0	$	C	1
1	C	G	4
2	C	$	0
3	C	T	5
4	G	T	6
5	T	C	2
6	T	C	3



A visual representation of LF for $T = \begin{array}{ccccccc} 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ C & T & C & T & G & C & \$ \end{array}$

Considering *LF* as an automaton

i	F	L	LF
0	\$	C	1
1	C	G	4
2	C	\$	0
3	C	T	5
4	G	T	6
5	T	C	2
6	T	C	3



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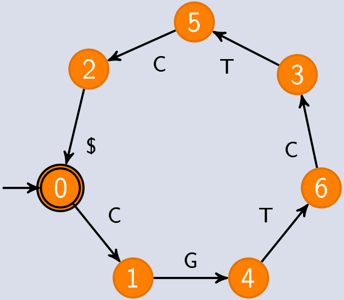
Considering *LF* as an automaton

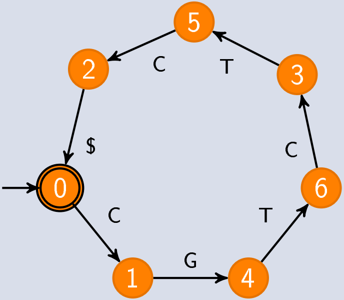
i	*F*	*L*	*LF*
0	\$	C	1
1	C	G	4
2	C	\$	0
3	C	T	5
4	G	T	6
5	T	C	2
6	T	C	3



A visual representation of *LF* for $T = \begin{array}{ccccccc} 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ \mathsf{C} & \mathsf{T} & \mathsf{C} & \mathsf{T} & \mathsf{G} & \mathsf{C} & \$ \end{array}$

Considering *LF* as an automaton

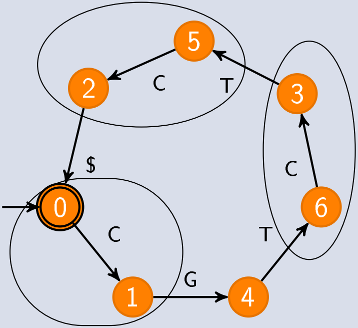
i	*F*	*L*	*LF*
0	$	C	1
1	C	G	4
2	C	$	0
3	C	T	5
4	G	T	6
5	T	C	2
6	T	C	3

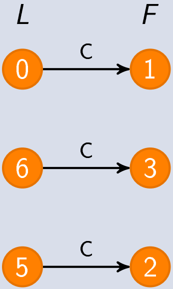


A visual representation of *LF* for $T = \overset{0}{C}\overset{1}{T}\overset{2}{C}\overset{3}{T}\overset{4}{G}\overset{5}{C}\overset{6}{\$}$

Considering *LF* as an automaton

i	*F*	*L*	*LF*
0	\$	C	1
1	C	G	4
2	C	\$	0
3	C	T	5
4	G	T	6
5	T	C	2
6	T	C	3

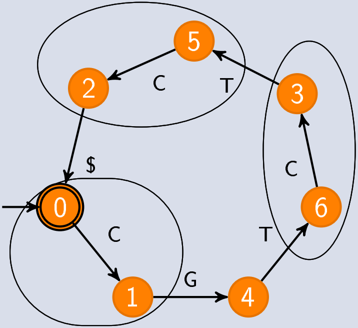


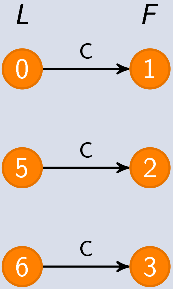


A visual representation of *LF* for $T = \overset{0}{C}\overset{1}{T}\overset{2}{C}\overset{3}{T}\overset{4}{G}\overset{5}{C}\overset{6}{\$}$

Considering *LF* as an automaton

i	*F*	*L*	*LF*
0	\$	C	1
1	C	G	4
2	C	\$	0
3	C	T	5
4	G	T	6
5	T	C	2
6	T	C	3

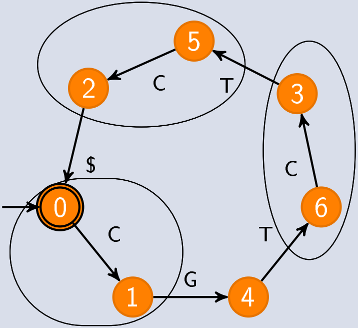


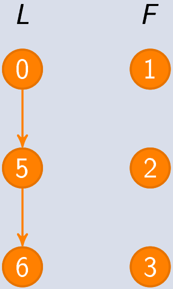


A visual representation of *LF* for $T = \overset{0}{\mathrm{C}}\,\overset{1}{\mathrm{T}}\,\overset{2}{\mathrm{C}}\,\overset{3}{\mathrm{T}}\,\overset{4}{\mathrm{G}}\,\overset{5}{\mathrm{C}}\,\overset{6}{\$}$

Considering *LF* as an automaton

i	*F*	*L*	*LF*
0	$	C	1
1	C	G	4
2	C	$	0
3	C	T	5
4	G	T	6
5	T	C	2
6	T	C	3

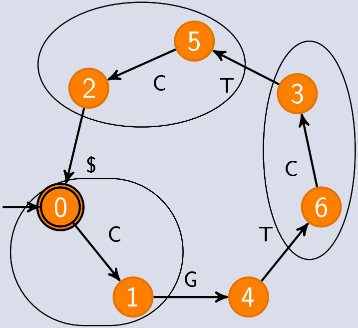


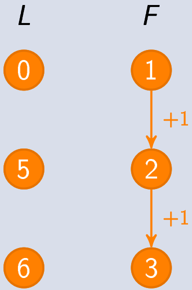


A visual representation of *LF* for $T = \overset{0}{C}\,\overset{1}{T}\,\overset{2}{C}\,\overset{3}{T}\,\overset{4}{G}\,\overset{5}{C}\,\overset{6}{\$}$

Considering *LF* as an automaton

i	*F*	*L*	*LF*
0	$	C	1
1	C	G	4
2	C	$	0
3	C	T	5
4	G	T	6
5	T	C	2
6	T	C	3

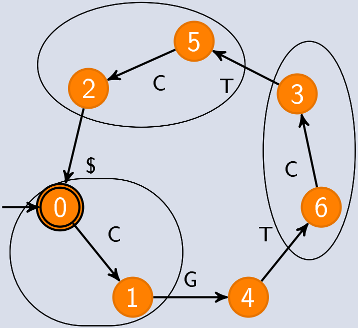


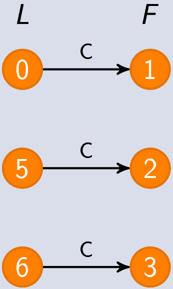


A visual representation of *LF* for $T = \overset{0}{\mathrm{C}}\,\overset{1}{\mathrm{T}}\,\overset{2}{\mathrm{C}}\,\overset{3}{\mathrm{T}}\,\overset{4}{\mathrm{G}}\,\overset{5}{\mathrm{C}}\,\overset{6}{\$}$

Considering *LF* as an automaton

i	*F*	*L*	*LF*
0	$	C	1
1	C	G	4
2	C	$	0
3	C	T	5
4	G	T	6
5	T	C	2
6	T	C	3





Property

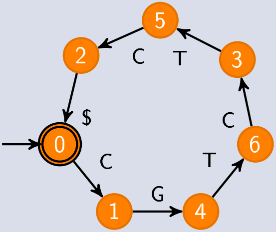
If $i_1 \xrightarrow{c} i_2$ $j_1 \xrightarrow{c} j_2$ then $i_1 < j_1 \iff i_2 < j_2$

Updating *LF* and the BWT

$$T = \begin{array}{ccccccc}0&1&2&3&4&5&6\\ C&T&C&T&G&C&\$\end{array} \rightarrow T' = \begin{array}{cccccccc}0&1&2&3&4&5&6&7\\ C&T&G&C&T&G&C&\$\end{array}$$

Stage 1: inserted letter *G* is between \$ and *L* (not in *L*)

i	*F*	*L*
0	\$	C
1	C	G
2	C	\$
3	C	T
4	G	T
5	T	C
6	T	C



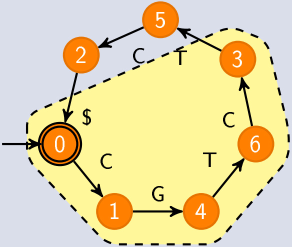
F							*L*
\$	C	T	*G*	C	T	G	C
C	\$	C	T	*G*	C	T	G
⋮							
G	C	\$	C	T	*G*	C	T
⋮							
T	G	C	\$	C	T	*G*	C

Updating *LF* and the BWT

$$T = \overset{0}{C}\,\overset{1}{T}\,\overset{2}{C}\,\overset{3}{T}\,\overset{4}{G}\,\overset{5}{C}\,\overset{6}{\$} \rightarrow T' = \overset{0}{C}\,\overset{1}{T}\,\overset{2}{\color{red}{G}}\,\boxed{\overset{3}{C}\,\overset{4}{T}\,\overset{5}{G}\,\overset{6}{C}\,\overset{7}{\$}}$$

Stage 1: inserted letter *G* is between $ and *L* (not in *L*)

i	*F*	*L*
0	$	C
1	C	G
2	C	$
3	C	T
4	G	T
5	T	C
6	T	C



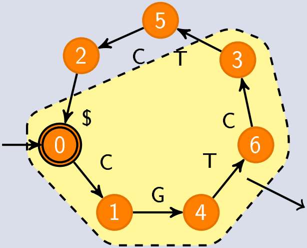
F							*L*
$	C	T	*G*	C	T	G	C
C	$	C	T	*G*	C	T	G
⋮							
G	C	$	C	T	*G*	C	T
⋮							
T	G	C	$	C	T	*G*	C

Updating LF and the BWT

$$T = \overset{0}{C}\overset{1}{T}\overset{2}{C}\overset{3}{T}\overset{4}{G}\overset{5}{C}\overset{6}{\$} \rightarrow T' = \overset{0}{C}\overset{1}{T}\overset{2}{G}\boxed{\overset{3}{C}\overset{4}{T}\overset{5}{G}\overset{6}{C}\overset{7}{\$}}$$

Stage 1: inserted letter *G* is between \$ and L (not in L)

i	F	L
0	\$	C
1	C	G
2	C	\$
3	C	T
4	G	T
5	T	C
6	T	C



Preserved zone

Best possible situation:

→ at position 1.

On average:

→ at position $n/2$.

Worst-case situation:

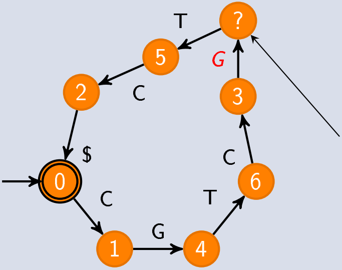
→ at position n.

Updating *LF* and the BWT

$$T = \overset{0}{C}\overset{1}{T}\overset{2}{C}\overset{3}{T}\overset{4}{G}\overset{5}{C}\overset{6}{\$} \to T' = \overset{0}{C}\overset{1}{T}\overset{2}{\color{red}G}\boxed{\overset{3}{C}\overset{4}{T}\overset{5}{G}\overset{6}{C}\overset{7}{\$}}$$

Stage 2: inserted letter *G* is in *L*

i	*F*	*L*
0	$	C
1	C	G
2	C	$
3	C	T
4	G	T
5	T	C
6	T	C



F   *L*

C T G C $ C T *G*

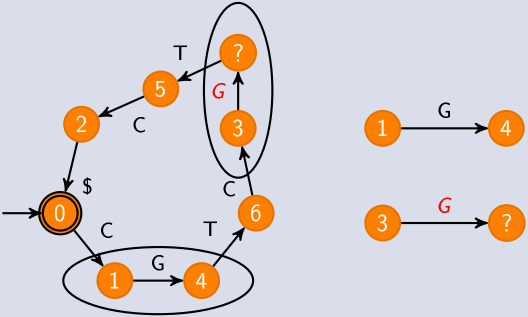
We have to consider all existing transitions labeled with *G*

Updating *LF* and the BWT

$$T = \overset{0}{C}\,\overset{1}{T}\,\overset{2}{C}\,\overset{3}{T}\,\overset{4}{G}\,\overset{5}{C}\,\overset{6}{\$} \rightarrow T' = \overset{0}{C}\,\overset{1}{T}\,\overset{2}{G}\,\overset{3}{C}\,\overset{4}{T}\,\overset{5}{G}\,\overset{6}{C}\,\overset{7}{\$}$$

Stage 2: inserted letter *G* is in *L*

i	*F*	*L*
0	$	C
1	C	G
2	C	$
3	C	*G*
4	G	T
5	T	C
6	T	C

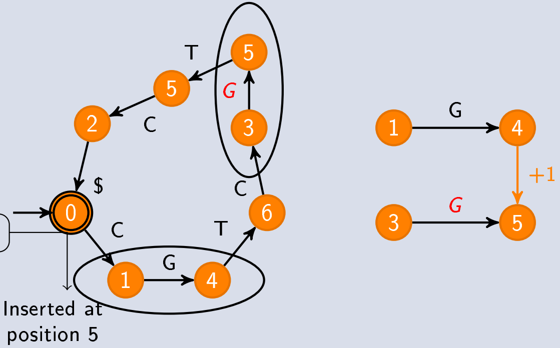


Updating *LF* and the BWT

$$T = \overset{0}{\mathrm{C}}\,\overset{1}{\mathrm{T}}\,\overset{2}{\mathrm{C}}\,\overset{3}{\mathrm{T}}\,\overset{4}{\mathrm{G}}\,\overset{5}{\mathrm{C}}\,\overset{6}{\$} \rightarrow T' = \overset{0}{\mathrm{C}}\,\overset{1}{\mathrm{T}}\,\overset{2}{\color{red}{G}}\,\overset{3}{\mathrm{C}}\,\overset{4}{\mathrm{T}}\,\overset{5}{\mathrm{G}}\,\overset{6}{\mathrm{C}}\,\overset{7}{\$}$$

Stage 2: inserted letter *G* is in *L*

i	*F*	*L*
0	$	C
1	C	G
2	C	$
3	C	*G*
4	G	T
5	*G*	T
6	T	C
7	T	C

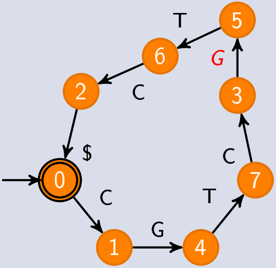


Updating *LF* and the BWT

$$T = \overset{0}{C}\,\overset{1}{T}\,\overset{2}{C}\,\overset{3}{T}\,\overset{4}{G}\,\overset{5}{C}\,\overset{6}{\$} \rightarrow T' = \overset{0}{C}\,\overset{1}{T}\,\overset{2}{\color{red}G}\,\overset{3}{C}\,\overset{4}{T}\,\overset{5}{G}\,\overset{6}{C}\,\overset{7}{\$}$$

Stage 2: inserted letter *G* is in *L*

i	*F*	*L*
0	$	C
1	C	G
2	C	$
3	C	*G*
4	G	T
5	*G*	T
6	T	C
7	T	C

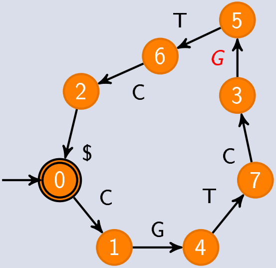


Updating *LF* and the BWT

$$T = \overset{0}{C}\,\overset{1}{T}\,\overset{2}{C}\,\overset{3}{T}\,\overset{4}{G}\,\overset{5}{C}\,\overset{6}{\$} \rightarrow T' = \overset{0}{C}\,\overset{1}{T}\,\overset{2}{G}\,\overset{3}{C}\,\overset{4}{T}\,\overset{5}{G}\,\overset{6}{C}\,\overset{7}{\$}$$

Stage 2: inserted letter *G* is in *L*

i	*F*	*L*
0	$	C
1	C	G
2	C	$
3	C	*G*
4	G	T
5	*G*	T
6	T	C
7	T	C



F							*L*
G	C	T	G	C	$	C	T

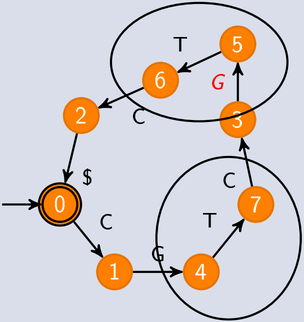
We have to consider all existing transitions labeled with T.

Updating *LF* and the BWT

$$T = \overset{0}{\mathrm{C}}\,\overset{1}{\mathrm{T}}\,\overset{2}{\mathrm{C}}\,\overset{3}{\mathrm{T}}\,\overset{4}{\mathrm{G}}\,\overset{5}{\mathrm{C}}\,\overset{6}{\$} \rightarrow T' = \overset{0}{\mathrm{C}}\,\overset{1}{\mathrm{T}}\,\overset{2}{G}\,\overset{3}{\mathrm{C}}\,\overset{4}{\mathrm{T}}\,\overset{5}{\mathrm{G}}\,\overset{6}{\mathrm{C}}\,\overset{7}{\$}$$

Stage 3: inserted letter *G* is in *F*: new row

i	*F*	*L*
0	\$	C
1	C	G
2	C	\$
3	C	*G*
4	G	T
5	*G*	T
6	T	C
7	T	C



Now, by T, we go to 6 from state 5 instead of 3.

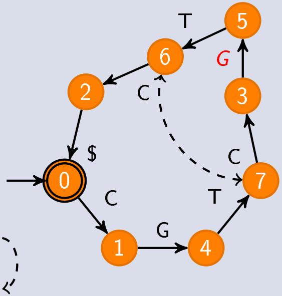
We check if the property is satisfied by considering the other edges labelled by T.

Updating *LF* and the BWT

$$T = \overset{0}{C}\,\overset{1}{T}\,\overset{2}{C}\,\overset{3}{T}\,\overset{4}{G}\,\overset{5}{C}\,\overset{6}{\$} \rightarrow T' = \overset{0}{C}\,\overset{1}{T}\,\overset{2}{G}\,\overset{3}{C}\,\overset{4}{T}\,\overset{5}{G}\,\overset{6}{C}\,\overset{7}{\$}$$

Stage 3: inserted letter *G* is in *F*: new row

i	*F*	*L*
0	\$	C
1	C	G
2	C	\$
3	C	*G*
4	G	T
5	*G*	T
6	T	C
7	T	C

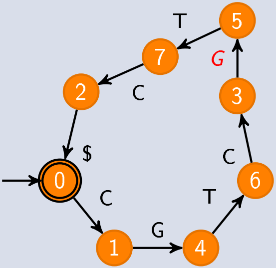


Updating LF and the BWT

$$T = \overset{0}{\mathrm{C}}\,\overset{1}{\mathrm{T}}\,\overset{2}{\mathrm{C}}\,\overset{3}{\mathrm{T}}\,\overset{4}{\mathrm{G}}\,\overset{5}{\mathrm{C}}\,\overset{6}{\$} \rightarrow T' = \overset{0}{\mathrm{C}}\,\overset{1}{\mathrm{T}}\,\overset{2}{G}\,\overset{3}{\mathrm{C}}\,\overset{4}{\mathrm{T}}\,\overset{5}{\mathrm{G}}\,\overset{6}{\mathrm{C}}\,\overset{7}{\$}$$

Stage 3: inserted letter G is in F: new row

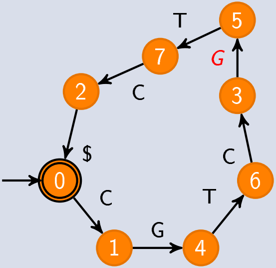
i	F	L
0	\$	C
1	C	G
2	C	\$
3	C	G
4	G	T
5	G	T
6	T	C
7	T	C



Updating *LF* and the BWT

$$T = \overset{0}{\mathrm{C}}\overset{1}{\mathrm{T}}\overset{2}{\mathrm{C}}\overset{3}{\mathrm{T}}\overset{4}{\mathrm{G}}\overset{5}{\mathrm{C}}\overset{6}{\$} \rightarrow T' = \overset{0}{\mathrm{C}}\overset{1}{\mathrm{T}}\overset{2}{G}\overset{3}{\mathrm{C}}\overset{4}{\mathrm{T}}\overset{5}{\mathrm{G}}\overset{6}{\mathrm{C}}\overset{7}{\$}$$

i	*F*	*L*
0	$	C
1	C	G
2	C	$
3	C	*G*
4	G	T
5	*G*	T
6	T	C
7	T	C



F *L*

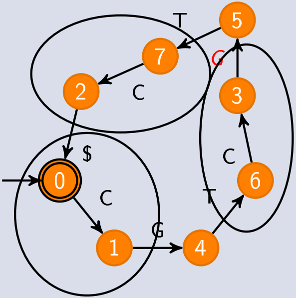
T *G* C T G C $ C

We have to consider all existing transitions labeled with C.

Updating *LF* and the BWT

$$T = \overset{0}{C}\overset{1}{T}\overset{2}{C}\overset{3}{T}\overset{4}{G}\overset{5}{C}\overset{6}{\$} \rightarrow T' = \overset{0}{C}\overset{1}{T}\overset{2}{G}\overset{3}{C}\overset{4}{T}\overset{5}{G}\overset{6}{C}\overset{7}{\$}$$

i	*F*	*L*
0	$	C
1	C	G
2	C	$
3	C	*G*
4	G	T
5	*G*	T
6	T	C
7	T	C

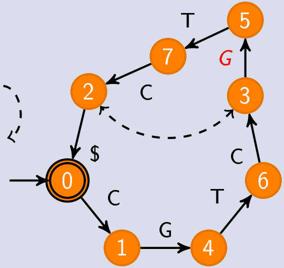


blablabla

Updating *LF* and the BWT

$$T = \overset{0}{\mathsf{C}}\,\overset{1}{\mathsf{T}}\,\overset{2}{\mathsf{C}}\,\overset{3}{\mathsf{T}}\,\overset{4}{\mathsf{G}}\,\overset{5}{\mathsf{C}}\,\overset{6}{\$} \rightarrow T' = \overset{0}{\mathsf{C}}\,\overset{1}{\mathsf{T}}\,\overset{2}{G}\,\overset{3}{\mathsf{C}}\,\overset{4}{\mathsf{T}}\,\overset{5}{\mathsf{G}}\,\overset{6}{\mathsf{C}}\,\overset{7}{\$}$$

i	*F*	*L*
0	$	C
1	C	G
2	C	$
3	C	*G*
4	G	T
5	*G*	T
6	T	C
7	T	C

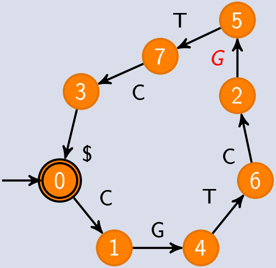


Same principle:
$6 < 7$ but $3 > 2$

Updating LF and the BWT

$$T = \overset{0}{C}\overset{1}{T}\overset{2}{C}\overset{3}{T}\overset{4}{G}\overset{5}{C}\overset{6}{\$} \rightarrow T' = \overset{0}{C}\overset{1}{T}\overset{2}{G}\overset{3}{C}\overset{4}{T}\overset{5}{G}\overset{6}{C}\overset{7}{\$}$$

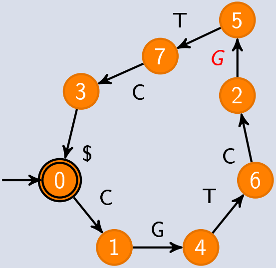
i	F	L
0	\$	C
1	C	G
2	C	G
3	C	\$
4	G	T
5	G	T
6	T	C
7	T	C



Updating *LF* and the BWT

$$T = \overset{0}{C}\,\overset{1}{T}\,\overset{2}{C}\,\overset{3}{T}\,\overset{4}{G}\,\overset{5}{C}\,\overset{6}{\$} \rightarrow T' = \overset{0}{C}\,\overset{1}{T}\,\overset{2}{G}\,\overset{3}{C}\,\overset{4}{T}\,\overset{5}{G}\,\overset{6}{C}\,\overset{7}{\$}$$

i	*F*	*L*
0	$	C
1	C	G
2	C	*G*
3	C	$
4	G	T
5	*G*	T
6	T	C
7	T	C



We have only one edge labelled $, therefore the property is satisfied.